1) (5 points) Suppose that $F : \mathbb{R}^n \rightarrow \mathbb{R}$ is continuously differentiable, and that $\mathbf{p}$ is a point in $\mathbb{R}^n$. What is the relationship between the level set where $F = F(\mathbf{p})$ and the gradient vector $\nabla F(\mathbf{p})$?

(Yes - you must write a sentence or two. Sentences start with capital letters, and end with some form of punctuation, typically a period.)

2) Let $F(x, y, z) = 2x^2 + 3y^3 - z$.

a) (4 points) The point $(1, 1, 2)$ lies on the level surface where $F = c$, if $c$ has what value?

b) (7 points) What is $\nabla F(1, 1, 2)$?

c) (7 points) Give an equation for the tangent plane to the level surface where $F = c$ ($c$ from part (a)) at the point $(1, 1, 2)$. 
3) Let \( \mathbf{r}(u, v) = (e^u \cos v, e^u \sin v, u) \).

a) (6 points) What are the values of \( \mathbf{r} \), \( \mathbf{r}_u \), and \( \mathbf{r}_v \) at the point \((0, \pi/2)\)?

b) (6 points) What is \( \mathbf{r}_u \times \mathbf{r}_v \) at the point \((0, \pi/2)\)?

c) (4 points) Is the parameterization \( \mathbf{r} \) regular at \((0, \pi/2)\)? How does your answer to (b) tell you this?

d) (4 points) Give a parameterization for the tangent plane of \( \mathbf{r} \) at \((0, \pi/2)\).

e) (4 points) Give an equation for the tangent plane of \( \mathbf{r} \) at \((0, \pi/2)\), which describes the tangent plane as a level set.
4) (15 points) Let \( f(x, y) = x^3 + 3xy - y^3 + 2 \). Verify that the critical points of \( f \) are non-degenerate, and classify each one as a point where \( f \) has a local maximum value, a local minimum value, or a saddle point.

5) (6 points) Suppose that \( g(x, y) \) has a non-degenerate critical point at \((1, 2)\), and attains a local minimum value at \((1, 2)\). Sketch what four level curves of \( g \) might look like near \((1, 2)\) and sketch in three possible gradient vectors of \( g \) on each level curve.
6) (16 points) Suppose that $x$ and $y$ are measured in meters. Let $E$ be the (compact) region in the $xy$-plane that is bounded between the graphs of $y = x^2$ and $y = 4$, including the top line-segment and the bottom parabolic edge. A heated metal plate occupies the region $E$. The temperature, in $^\circ$C, at a point $(x, y)$ on the plate is given by $T(x, y) = x(y - 1)$. Find the maximum and minimum temperatures on the plate.
7) (16 points) Use Lagrange multipliers to find all of the critical points of \( f(x, y) = x - y^2 \), subject to the constraint \( x^2 + y^2 = 1 \).
EXTRA CREDIT:

Recall that, in problem (3), we looked at the parameterization \( \mathbf{r}(u, v) = (e^u \cos v, e^u \sin v, u) \).

a) (3 points) Show that \( \mathbf{r} \) parameterizes (part of, or all of) the surface where \( x^2 + y^2 = e^{2z} \).

b) (2 points) Think about the \( z \)-cross sections of the surface where \( x^2 + y^2 = e^{2z} \), and how they change as \( z \) goes from \(-\infty\) to \( \infty \). Now sketch the surface where \( x^2 + y^2 = e^{2z} \), i.e., sketch the surface parameterized by \( \mathbf{r} \).